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Large-Scale Motion in the Upper Stratosphere and Mesosphere: An Evaluation of Data and Theories

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Large-scale motion in the upper stratosphere and mesosphere: an evaluation of data and theories

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1. Introduction

Between heights of 25 and 100 km, fluid motion is responsible for carrying energy from the radiative heat source near the summer pole to the radiative heat sink near the winter pole. The way this transfer is organized will form the theme of this contribution.

All scales of motion are important:

(1) The zonal-mean motion contains most of the kinetic energy but contributes to the energy transfer only through a comparatively weak circulation in the meridional plane.

(2) Stationary large-scale eddies are prominent at least in the winter and carry a considerable fraction of the energy transfer.

(3) Transient large-scale eddies of a variety of types are evident in the data but are likely to be feeble transporters of energy.

(4) Gravity-inertial waves, generated by tidal forcing and orography and representing some aspects of small-scale turbulence, are associated mainly with the vertical transfer of energy.

2. Zonal-mean flow and the energy transfer due to the mean meridional circulation

The zonal-mean flow is unable to carry the global scale heat flux because of the angular momentum it must also carry.

Let $\phi = (C_v/C_p) \ln p - \ln \rho$. Then $Q = \mathrm{D}\phi/\mathrm{D}t$ is the diabatic heating. Dividing this into radiative (Q_r) and eddy (Q_e) contributions the zonally averaged thermodynamic equation becomes

$$\frac{\partial \overline{\phi}^x}{\partial t} + \overline{w}^x \frac{\partial \overline{\phi}^x}{\partial z} = Q = Q_r + Q_e, \qquad (1)$$

where x is directed W–E, y is S–N, z upwards; u, v, w are corresponding components of velocity and the overbar denotes an average with respect to the suffix variable. The term in $v\partial\phi/\partial y$ has been neglected.

Similarly, let τ be the stress due to eddy and molecular processes. The zonal-mean zonal component of the momentum equation becomes

$$\frac{\partial \overline{u}^x}{\partial t} - f\overline{v}^x = \frac{1}{\rho}\frac{\partial \tau_{xz}}{\partial z}, \qquad (2)$$

where the terms in $v\partial u/\partial y$, $w\partial u/\partial z$ and $\partial \tau_{xy}/\partial y$ have been neglected. Consider if the mean meridional circulation can carry the heat flux without the help of the eddy transfer Q_e, as assumed by Murgatroyd & Singleton (1961) and Leovy (1964). The first term in equation (1) is observed to be much smaller than Q_r so $\overline{w} \simeq Q_r/B$ where $B = \partial\overline{\phi}^x/\partial z \simeq 4 \times 10^{-5}\ \mathrm{m}^{-1}$ and $Q_r \simeq 0.02\ \mathrm{day}^{-1}$

(equivalent to a cooling rate of 5 °C/day) giving $\overline{w} = 0.5\,\mathrm{cm\,s^{-1}}$. Continuity of mass demands $\bar{v} = 2\,\mathrm{m\,s^{-1}}$, both too small to be measured directly. However, a meriodinal velocity of this magnitude gives a term of 20 $\mathrm{m\,s^{-1}}$ per day in equation (2) which is far larger than $\partial u/\partial t$ and must therefore be balanced by the stress. This is the scheme considered by Leovy, and demands $\tau_{xz}/\rho \simeq 1.5\,\mathrm{m^2\,s^{-1}}$ or, expressed in terms of an eddy diffusivity k, $k = 10^3\,\mathrm{m^2\,s^{-1}}$.

Hines estimates $k = 300\,\mathrm{m^2\,s^{-1}}$ which is somewhat too small and which may be an overestimate for the purpose of this calculation. This is because the turbulence to which it refers is essentially of small vertical scale. Its energy is derived through the instability of the small-scale shears induced by gravity waves, and it may be unable to detect (and therefore diffuse) the variations of velocity on the larger scale we are concerned with here.

For example, the Richardson number may approach unity over layers as much as 1 km deep hence the flow may be subject to shearing instability on that scale, whereas over layers 20 km deep Ri is about 400 and the flow is very stable. Again, if the turbulence were such as to mix momentum and potential temperature similarly, the turbulent flux of energy in the vertical would be considerable: leading to apparent heating rates of about 10 °C per day everywhere.

Alternatively however, shearing instability in shallow layers may generate gravity waves which propagate their influence to the more distant levels. This picture is attractive because such motion can transfer momentum (through pressure forces) more readily than energy (through mixing). However, propagating modes tend to maintain their momentum flux independent of height (rather than pick up momentum at one level and deliver it to another as required here) and so tend to be rather inefficient redistributors of momentum. Supposing that the estimates of turbulent intensity refer to this process and that it has an efficiency of 30 %, an apparent k of 100 $\mathrm{m^2\,s^{-1}}$ is obtained.

Thus values of k for momentum transfer over tens of kilometres probably lie in the range: $300 > k > 100\,\mathrm{m^2\,s^{-1}}$, implying a meridional circulation which could carry only between a third and a tenth of the mid-latitude heat flux. Clearly the actual transfer must be by a large-scale eddy process.

Incidentally, if the terms in $\partial/\partial t$ in equations (1) and (2) are negligible, ϕ satisfies a simple, unexpected, relation. Continuity of mass demands a relation between τ and Q and if, in addition ρk varies slowly with z (as would be expected for propagating modes at least) we can readily show that ϕ satisfies a diffusion equation; $Q = K\partial^2\phi/\partial y^2$ in which $K = (gB/f^2)\,k$. The values of K corresponding to the above limits on k are $(1.0 > K > 0.3 \times 10^7\,\mathrm{m^2\,s^{-1}})$. This gives the same estimate for the meridional heat transfer as the above discussion, but the relation is novel and we can see more clearly how the meridional circulation becomes more powerful in the equatorial regions where $f \to 0$.

3. Stationary eddies

As pointed out by Charney & Drazin (1961) stationary eddies can be propagated upwards, depending on the nature of a frequency equation similar to that for stationary Rossby waves. If

$$\boldsymbol{v}_{\mathrm{e}} = A\rho^{-\frac{1}{2}} \exp \mathrm{i}(s\,\mathrm{long} + \nu z)\, P_n^s(\mathrm{lat}) \quad \text{(eddy velocity)}$$

then

$$\nu^2 = \frac{gB}{f^2}\left(\frac{\beta}{\overline{u}^y} - \frac{n(n+1)}{a^2}\right) - \frac{1}{4H_0^2} \tag{3}$$

where $\beta = \partial f/\partial y$, a = Earth's radius, H_0 = density scale height, and $n \geqslant s$. From the expression for $\boldsymbol{v}_{\mathrm{e}}$ we see that specific kinetic energy $\frac{1}{2}\rho \boldsymbol{v}_{\mathrm{e}}^2$ can be propagated upwards only if ν^2 is positive, in which case the phase of the motion changes systematically with z. If ν^2 is negative $\frac{1}{2}\rho v_e^2$ decreases upwards and the phase is constant. In the intermediate layer for which $0 > \nu^2 > -1/4H_0^2$, $\frac{1}{2}\rho v_e^2$ decreases but the wave amplitude increases upwards.

Putting $a^2f^2/4gBH_0^2 = 6$, the criterion for no reflexion of energy becomes:

$$0 < \bar{u}^y < 600\ \mathrm{m\ s^{-1}}/\{n(n+1)+6\} \equiv U_1 \tag{4}$$

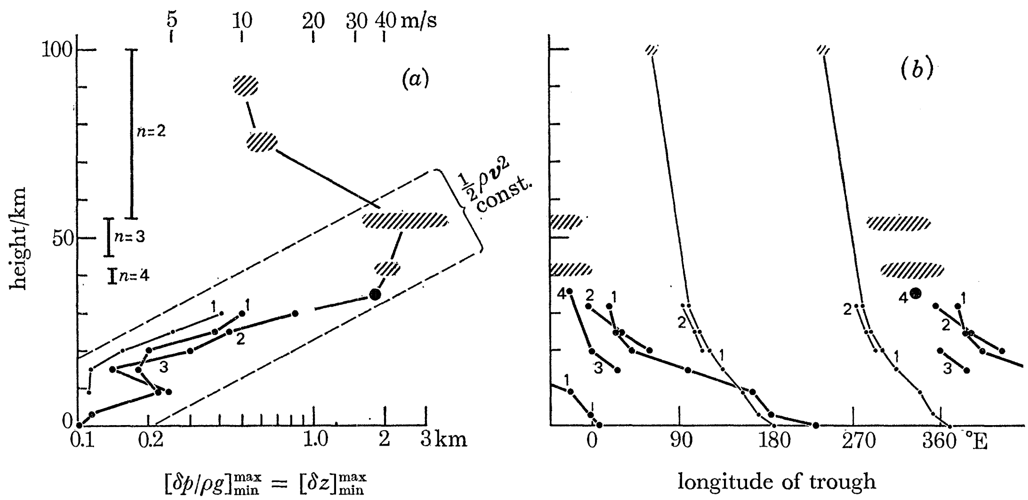

FIGURE 1. Variation with height of (*a*) the amplitude and (*b*) the phase of the January mean eddy motion at 50° N. (*a*) Amplitude, defined as the difference between maximum and minimum contour height is plotted on a logarithmic scale. Where the specific kinetic energy is constant the variation is parallel to the thin broken line. Also shown are levels where, for several values of n, $\frac{1}{2}\rho v_e^2$ should begin to decrease upwards (lower limit) and where the amplitude $|v_e|$ should begin to decrease upwards (upper limit). (*b*) Phase, defined as longitude of trough. Thick line, $s = 1$; thin line, $s = 2$; undetermined component shaded. For origin of data see §3.1.

and the criterion for the wave amplitude increasing upwards, but not the kinetic energy is

$$U_1 < \bar{u}^y < 600\ \mathrm{m\ s^{-1}}/n(n+1) \equiv U_2. \tag{5}$$

These two limits are shown in figure 1. Later, more refined analyses, e.g. Dickinson (1968) support the general conclusions.

3.1. *Origin of data for figure* 1

For the monthly mean motion data coverage is quite good up to the 10 mbar† (30 km) level. Curve *1* is for January 1958 from Muench (1965), *2* is for January 1966 from Hirota & Sato (1969), *3* is calculated for the January averages over a number of years taken from *Geophysical memoirs* 112 and 103.

Between 5 and 0.4 mbar (35 and 55 km) the upward extrapolation of pressure is based on infrequent rocket soundings mostly over the North American continent. Point 4 is calculated for January 1964 from the charts in Essa technical report WB–2, whereas the values at 2 and 0.4 mbar are estimated by eye from the same data – there being insufficient coverage to allow a harmonic analysis.

Between 60 and 80 km data, in the form of rocket winds is even more sparse and the point at 75 km is based partly on Newell, Wallace & Mahoney (1966) partly on Webb (1966).

Between 80 and 100 km the amplitude comes from the Adelaide and Jodrell Bank data

† 1 mbar = 10^2 Pa.

given by Greenhow & Neufeld (1961) and the phase from Dr Katasyev's most valuable coverage of meteor-trail drifts shown at this meeting.

3.2. *Interpretation of data*

Figure 1 shows an analysis of the data readily available in terms of the amplitude and phase of the larger scale motion. The lowest eddy mode is $s = 1$ (representing an off-pole circulation) and contains components for which $n \geqslant 1$. The next lowest mode $s = 2$ contains components for which $n \geqslant 2$. Unfortunately the data, even when complete, has been analysed only into its zonal (or s) components, whereas in order to interpret the theory we need particularly the meridional (or n) resolution. Even so the data can be interpreted usefully.

Evidently there is complete energy transmission between the lower stratosphere (15 km) and 30 to 40 km where the modes for which $n \geqslant 3$ should be reflected according to the theory. (Indeed if anything there appears to be a small *increase* in the energy.)

Between 50 and 80 km there is a substantial decrease in amplitude which can be consistent with the theory only if there is remarkably little energy in the $n = 2$ mode whose amplitude should still be increasing at these levels.

The phase pattern is roughly consistent with the amplitude pattern. The $s = 1$ mode shows a systematic tilt of phase towards the west with increasing height at least up to 50 km, consistent with energy transmission in this layer. Similarly, the phase of the $s = 2$ mode slopes (and more steeply as it should) below 40 km where energy is transmitted, becoming constant above, where energy is reflected.

A more even distribution of observations would certainly improve the quality of such analyses, but already we can see, on extrapolating back to the surface along a line of constant kinetic energy, that remarkably little energy reaches the upper levels. This contrasts with later theories seeking new routes for injecting even more energy.

3.3. *Energy flux due to stationary eddies: forced ventilation of the winter pole*

Because the thermal wind relation is closely satisfied by the actual wind, the energy flux can be expressed simply in terms of the amplitude and phase of the velocity field. Suppose that at some latitude,

$$v = A \cos s(l - l_0) \quad \text{where} \quad l = \text{longitude}.$$

The amplitude A and phase l_0 may vary with z and the thermal wind relation:

$$\frac{\partial v}{\partial z} = \frac{g}{af \cos \varphi} \frac{\partial \phi}{\partial l} \quad \text{requires} \quad \phi - \overline{\phi}^x = \frac{af \cos \varphi}{g} \left\{ \frac{1}{s} \frac{\partial A}{\partial z} \sin s(l - l_0) - A \frac{\partial l_0}{\partial z} \cos s(l - l_0) \right\}$$

whence

$$\overline{v\phi}^x = -\frac{af \cos \varphi}{2g} A^2 \frac{\partial l_0}{\partial z} \quad \text{where } \varphi \text{ is latitude.} \tag{6}$$

Putting $\partial l_0/\partial z = -1/20$ km, $A = 30$ m s^{-1} as typical of 50° N between heights of 40 and 60 km in January gives a temperature variation of ± 20 °C and $\overline{v\phi} = 0.7$ m s^{-1}. This is equivalent to a heating rate of 9 °C/day everywhere poleward of 50° N; which is considerably more than can be supplied by the meridional circulation and is sufficient to carry the whole mid-latitude transfer.

4. Transient eddies

Apart from tidal variations there are transient velocities of about 10 m s^{-1} in the summer and 30 to 40 m s^{-1} in the winter. What is the origin of these disturbances and what do they do?

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Transient waves of tropospheric origin cannot penetrate to these heights so the motion must be local in origin: it is most probably the result of baroclinic instability.

4.1. *Mesospheric waves*

Large-scale waves usually grow near a rigid horizontal surface bounding a baroclinic region. The mesosphere is only a layer of lesser static stability but a slight generalization of the argument of Green (1960) shows that waves can grow on the 'unrigid lid' separating it from the more stable layers above (if the shear is easterly) and below (if the shear is westerly). These conditions are both satisfied in the winter when for uncritical values of the parameters the most unstable wave has a length of 4000 km and steering level some 2 km into the mesosphere, which means that both waves have a phase speed of about 50 m s^{-1} and period relative to the surface of 2 days. The waves are limited in vertical extent and therefore contribute little to the total energy transfer.

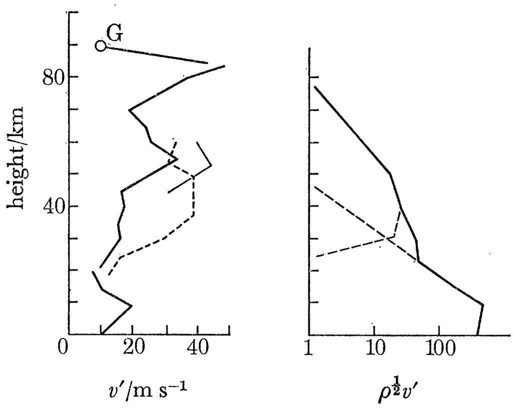

FIGURE 2. Variation with height of the transient velocities. Logarithmic plot on right allows tropospheric effect to be extrapolated upwards and substracted: dashed line, showing maximum of transient kinetic energy near 40 km. Data from Newell *et al.* (1966). Greenhow & Neufeld (1961) and Appleman (1963).

The transient velocities shown in figure 2 are large near 55 and 85 km which is consistent with being due to such a mechanism. It would be interesting to observe if these maxima were associated with periods of a few days, and the fluctuations observed by Dr Muller (this volume, p. 585) may be relevant.

4.2. *Stratospheric waves: spring warming*

In addition there may be eddies in the 20 to 60 km layer. Putting $\partial u/\partial z = 1.7\times10^{-3}\,s^{-1}$, $\beta = 1.2\times10^{-11}\,m^{-1}\,s^{-1}$, $B = 5\times10^{-5}\,m^{-1}$ gives a most unstable wave of length 4000 km with steering level near 30 km so a phase speed of 5 m s^{-1} and period relative to the surface of about a week. Removing the tropospheric effect from figure 2 (by extrapolating along the lower dashed line) leaves a distinct peak in specific kinetic energy near a height of 40 km which agrees well. Thus we suppose that, contrary to the conclusion of Murray (1960) the stratosphere may be baroclinically unstable.

Calculations of the amplitude and efficiency of baroclinic waves in the troposphere can be used to estimate the energy flux. These calculations give (Green 1970) eddy velocities of magnitude V where:

$$V^2 = g(\Delta\phi)^2/12B \quad \text{so} \quad V \sim 15\ \mathrm{m\,s^{-1}}$$

and

$$\overline{v\phi} = \alpha\,(g/B)^{\frac{1}{2}}\,(\Delta\phi)^2 \quad \text{so} \quad \overline{v\phi} \sim 0.03\ \mathrm{m\,s^{-1}},$$

where $\Delta\phi$ is the total variation of ϕ in the horizontal, and we have used $\alpha = 0.006$; i.e. an empirical value determined for the troposphere. We conclude that in the upper atmosphere the transient eddies transfer little energy compared to the stationary eddies: the opposite way round to the situation in the troposphere.

In conclusion, the mid-latitude transfer is by stationary eddies, the low-latitude transfer by the meridional circulation. The sense and magnitude of the weak meridional circulation in high latitudes depends on the intensity and distribution of the eddy transfer.

5. Tidal motion

Theory and observation have recently been reviewed by Chapman & Lindzen (1970), but two features are worthy of discussion.

5.1. *Energy transfer implies momentum transfer*

If, as seems plausible, energy is propagated upwards there must be a correlation between pressure and vertical velocity. But pressure and the zonal component of velocity are also correlated so a non-zero energy flux also implies a non-zero momentum flux. Now energy can be converted into heat and radiated away but angular momentum is indestructable and would steadily accumulate. Nor would the rate of accumulation be all that slow. I estimate that

$$|u|/|w| \simeq 1.5\,\omega^2 a/g \quad \text{so} \quad \overline{uw} \simeq -2\times 10^{-3}\,u^2,$$

giving an equivalent velocity tendency at 80 km of 3 m s^{-1} per day. If this effect is genuine it poses an interesting speculation on how the angular momentum (possibly redistributed by a meridional circulation) is balanced. Notice that the effect is basically independent of the variation of zonal velocity with height and so is different from the *internal* transfer of momentum discussed in §2.

5.2. *The 24 h tide as a messenger between troposphere and ionosphere*

Because the 24 h tide has a very short vertical wavelength it can be excited only by diabatic heating in the troposphere and consists almost entirely of a propagating mode above. Since the vertical wavelength is small it is susceptible to viscous dissipation in a shallow layer at a height of about 100 km. Now there is much evidence that this component is rather variable, depending on local synoptic conditions and it may be that this variability is transmitted to the upper levels by this process of energy transfer and dissipation. The observations quoted by Dr Muller (this volume, p. 585) may indeed be an example and the possible connexion might be explored further by analysing the tidal components over the period in question.

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